1. Holes

Factor First! Any factors that are in both the numerator and denominator create a hole. Set those factors equal to 0 and solve.

Example: $f(x) = \frac{x^3 - x}{-4x^2 + 4x} = \frac{x(x+1)(x-1)}{-4x(x-1)}$

Since the *x* and the (x – 1) factors are in the numerator and denominator, they will divide to be 1. Set both equal to zero and solve.

Answer: Holes at x = 0 and x = 1

2. Vertical Asymptotes

Any factors that in the denominator that did not create a hole, create a vertical asymptote. These represent all other values of x that make the function undefined.

Example: $f(x) = \frac{-4x}{x^2-x} = -\frac{4x}{x(x-1)}$

While there is a hole at x = 0, there is a vertical asymptote at x = 1.

Answer: VA: x = 1

3.Domain

Recall from simplifying rational expressions that anything that makes the denominator 0 is a domain restriction. In this case, anything that causes a hole or creates a vertical asymptote violates the domain.

Example: $f(x) = \frac{-4x}{x^2-x} = -\frac{4x}{x(x-1)}$

There is a hole at x = 0 and there is a vertical asymptote at x = 1. Therefore, both 0 and 1 are domain restrictions.

Answer: Domain: $x \neq 0, 1$

4. Horizontal Asymptote

Horizontal Asymptotes describe what is happening as the x-values in a function go toward positive or negative infinity. This is similar to end behavior that we discussed before. The value a function approaches depends on the degrees of the numerator and denominator.

- If the degree of the numerator is higher → none
- If the degree of the denominator is higher → y = 0
- If the degrees are the same → $y = \frac{leading\ coefficient\ of\ numerator}{leading\ coefficient\ of\ the\ denominator}$

Example: $f(x) = \frac{-4x}{x^2-x} = -\frac{4x}{x(x-1)}$

The degree of the numerator is 1 and the degree of the denominator is 2 (look at the unfactored form).

Answer: HA: y = 0

5. x-intercept(s)

x – Intercepts occur when the numerator is equal to 0. To find them, set the factored form of your numerator equal to zero and solve. Be aware of your domain restrictions! You cannot have an intercept at that spot if it violates the domain. Also, write your answer as an ordered pair.

Example: $f(x) = \frac{x^3 - x}{-4x^2 + 4x} = \frac{x(x+1)(x-1)}{-4x(x-1)}$

While I get three x-intercepts based on the numerator alone, 2 of them violate the domain. I cannot plug in x = 0 or x = 1.

Answer: x – intercept: (-1, 0)

6. y-intercept

The y – intercept occurs when x = 0. To find it, evaluate both the numerator and denominator for x = 0. Be aware of your domain restrictions! You cannot have an intercept at that spot if it violates the domain. Also, write your answer as an ordered pair.

Example: $f(x) = \frac{x^3 - x}{-4x^2 + 4x} = \frac{x(x+1)(x-1)}{-4x(x-1)}$

This example would not have a y-intercept because when I plug in x = 0, the denominator becomes zero. Note: The numerator is allowed to be zero.

Answer: y – intercept: *none*